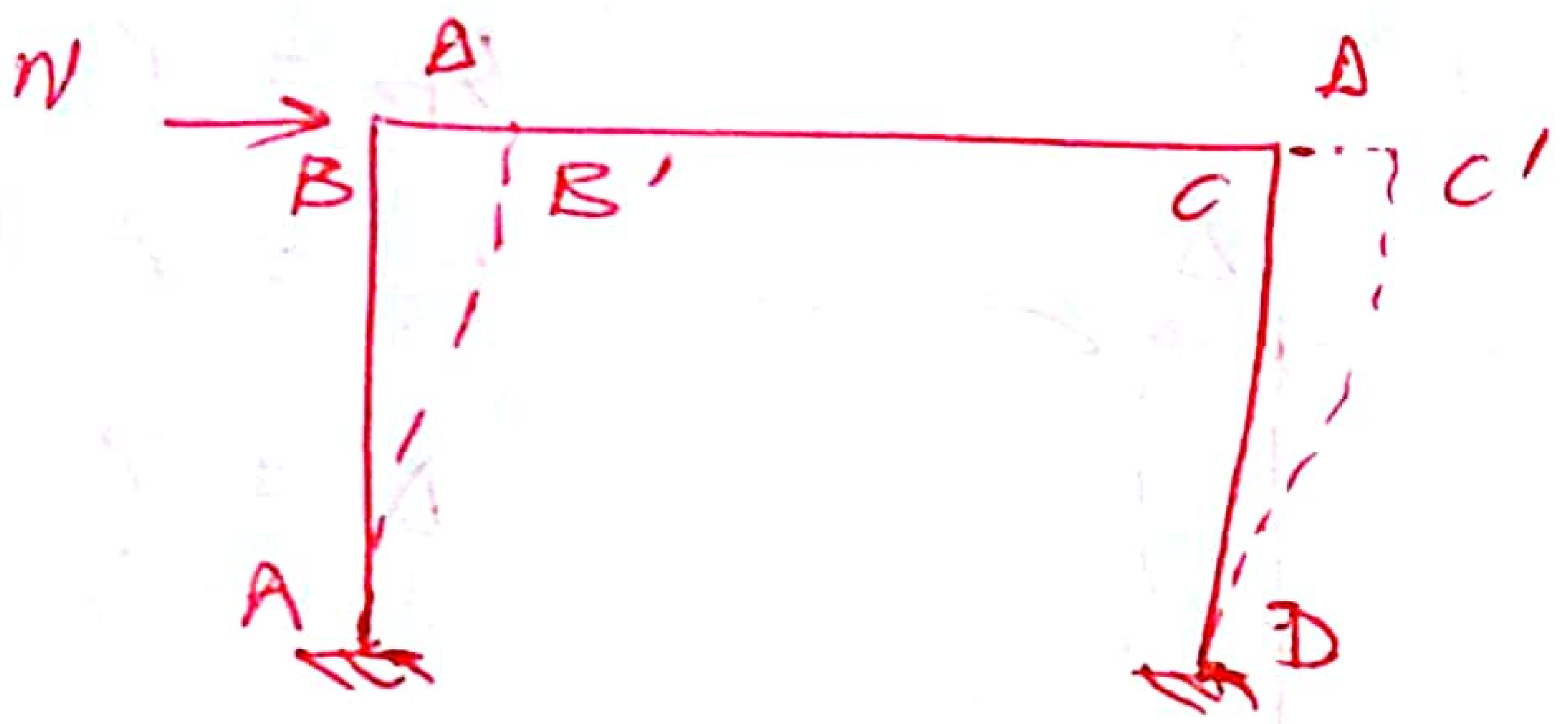
Analysis of Frames

- With Sway.
- Without Sway.

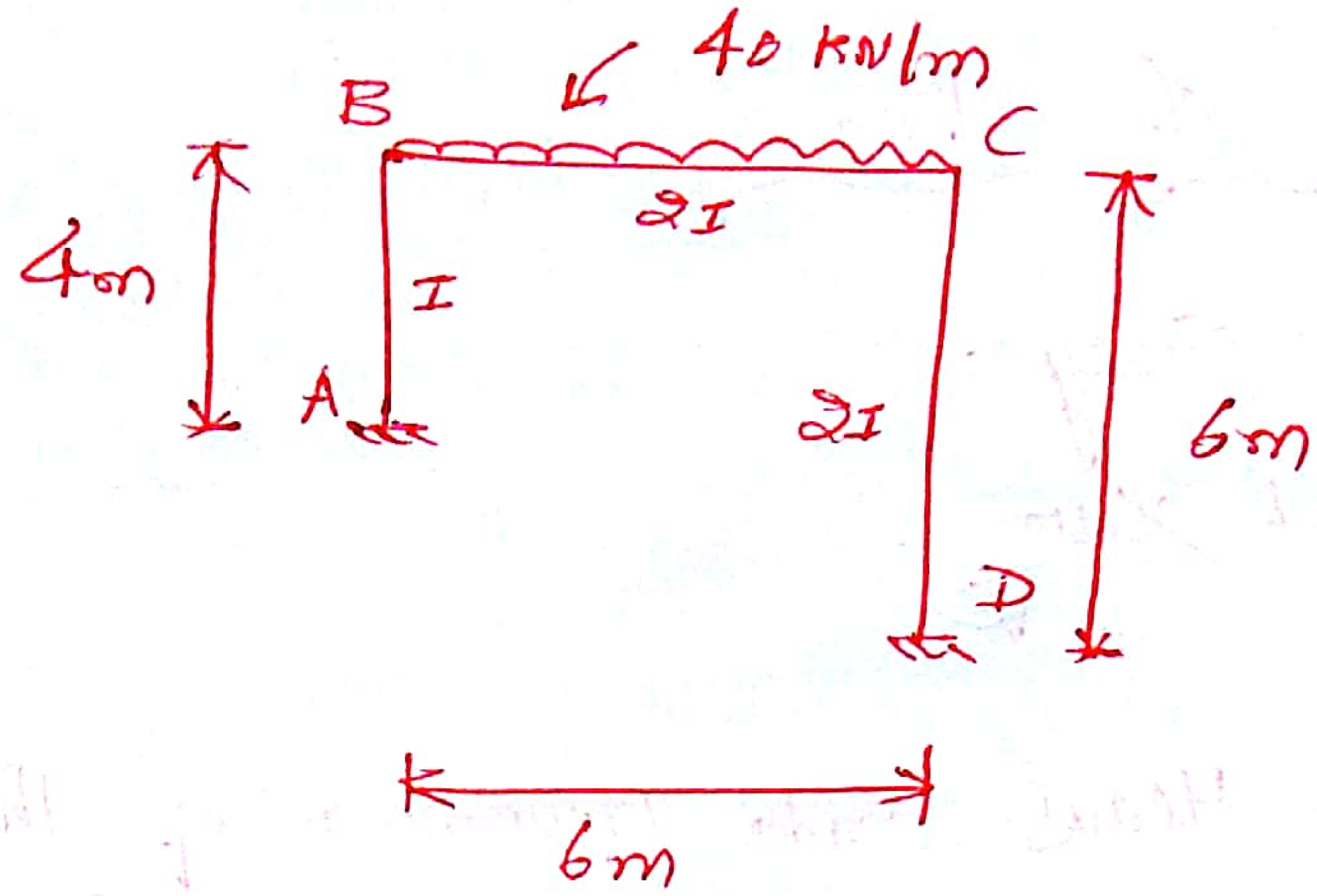
~~Sw~~ Sway – Means Side Movement of the End of a Column in a frame



Here, $\Delta \Rightarrow$ Sway.

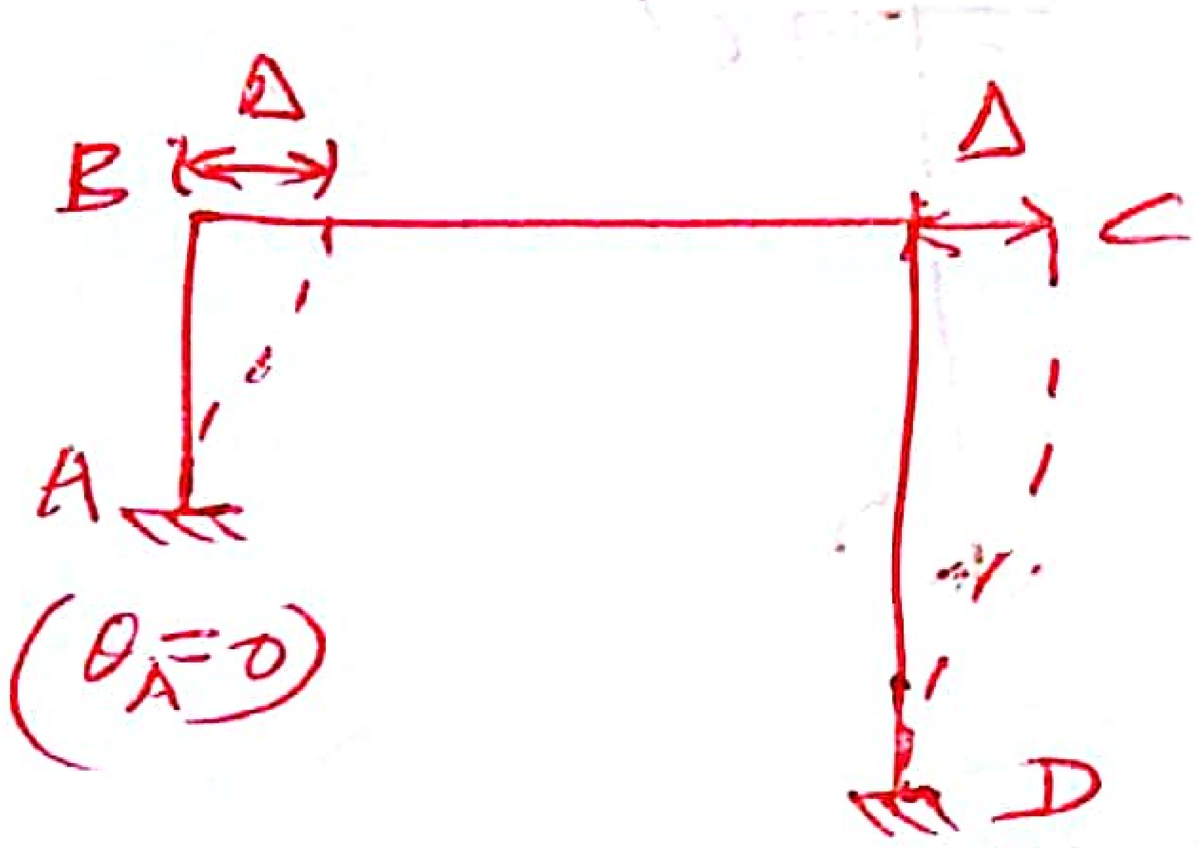
* For frames without sway same Procedure is applied as that for continuous Beams.

Quest →



(Sway conditions)

Sol →



$(\theta_A = 0)$

$(\theta_D = 0)$

$\theta_B = ?$

$\theta_C = ?$

$\Delta = ?$

→ 3 unknowns

∴ 3 eqns. are required

Step-1

- Fixed End Moments.

Here Note that $(M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0)$

No Fixed End Moments Acting as there is no load Present.

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{40 \times 6^2}{12} = -120 \text{ kNm}$$

$$M_{FCB} = \frac{wl^2}{12} = 120 \text{ kNm}$$

Step - 2

- slope deflection Equations

{* Here Note that $\left(\frac{3\Delta}{4}\right)^{*}$ will Be Included for AB and CD}

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\cancel{\theta_A}^{0} + \theta_B - \frac{3\Delta}{4}\right)$$

$$\boxed{M_{AB} \Rightarrow \frac{2EI}{4}\theta_B - \frac{6\Delta EI}{4}}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}\left(\cancel{\theta_A}^{0} + 2\theta_B - \frac{3\Delta}{4}\right)$$

$$\boxed{M_{BA} = EI\theta_B - \frac{3\Delta EI}{8}}$$

$$M_{BC} = -120 + \frac{2E(2I)}{6}\left(2\theta_B + \theta_C\right)$$

{Here 'Δ' term is Not Included B'coz Axial deformation of BC is assumed negligible}

$$\boxed{M_{BC} = -120 + \frac{4EI}{3}\theta_B + \frac{4EI}{6}\theta_C}$$

$$M_{CB} = 120 + \frac{2E(2I)}{6}\left(2\theta_C + \theta_B\right)$$

$$\boxed{M_{CB} = 120 + \frac{4EI}{3}\theta_C + \frac{4EI}{6}\theta_B}$$

$$M_{CD} = 0 + \frac{2E(2I)}{6}\left(2\theta_C + \cancel{\theta_D}^{0} - \frac{3\Delta}{6}\right)$$

$$\boxed{M_{CD} = \frac{4EI}{3}\theta_C - \frac{\Delta EI}{3}}$$

$$M_{DC} = 0 + \frac{2E(2I)}{6}\left(\cancel{2\theta_D}^{0} + \theta_C - \frac{3\Delta}{6}\right)$$

$$\boxed{M_{DC} = \frac{4EI}{6}\theta_C - \frac{\Delta EI}{3}}$$

STEP-3

- Equilibrium equations

$\Sigma M_B = 0$	$\Sigma M_C = 0$	Horizontal Equilibrium ($H_A + H_D = 0$)
(1)	(2)	(3)

$\Sigma M_B = 0$;

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B + \frac{3\Delta EI}{8} + \left(-120 + \frac{4EI}{3}\theta_B + \frac{4EI}{6}\theta_C\right) = 0$$

$$\left(\frac{7EI\theta_B}{3} + \frac{4EI\theta_C}{6} - \frac{3\Delta EI}{8} - 120 = 0\right)$$

$\Sigma M_C = 0$;

$$M_{CB} + M_{CD} = 0$$

$$120 + \frac{4EI}{3}\theta_C + \frac{4EI}{6}\theta_B + \frac{4EI}{3}\theta_C - \frac{\Delta EI}{3} = 0$$

$$\left(\frac{4EI\theta_B}{6} + \frac{8EI\theta_C}{3} - \frac{\Delta EI}{3} + 120\right) = 0$$

Taking Moments About B and C

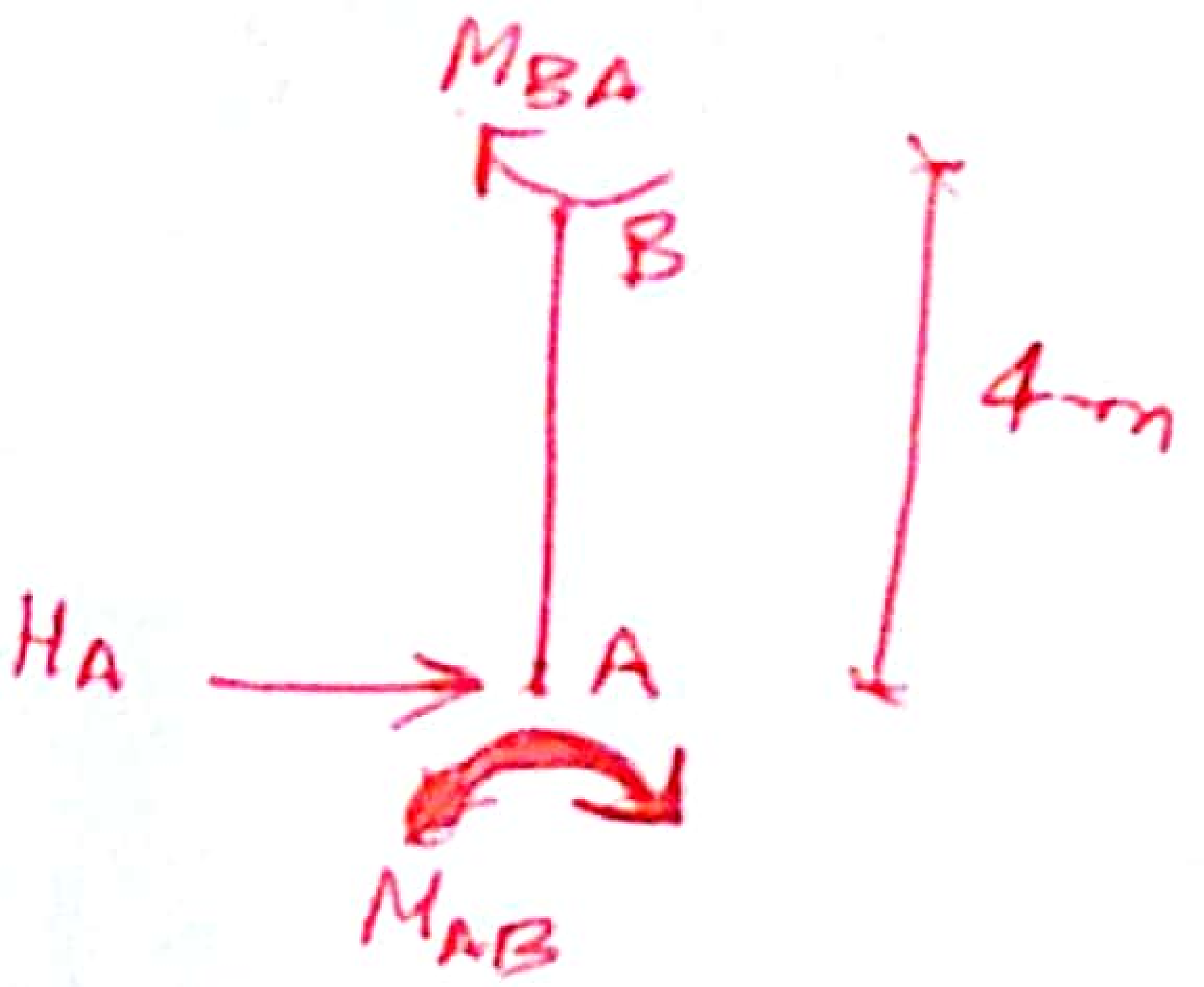


Fig: For AB

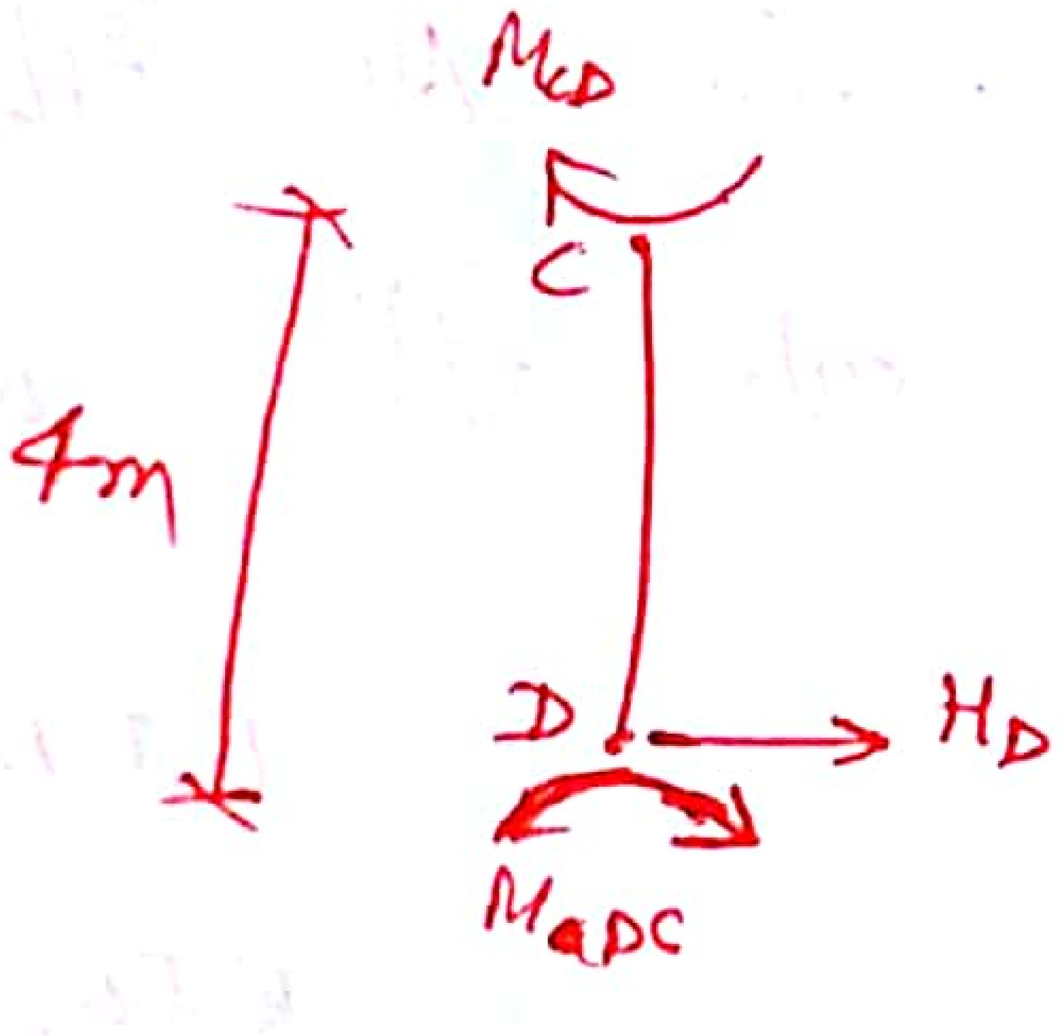


Fig: For CD

$$(-H_A \times 4) + M_{AB} + M_{BA} = 0$$

$$+H_A = +\left(\frac{M_{AB} + M_{BA}}{4}\right)$$

$$\left(H_A = \frac{M_{AB} + M_{BA}}{4}\right)$$

$$(-H_D \times 4) + M_{CD} + M_{DC} = 0$$

$$+H_D = +\left(\frac{M_{CD} + M_{DC}}{4}\right)$$

$$\left(H_D = \frac{M_{CD} + M_{DC}}{4}\right)$$

Horizontal Equilibrium :

$$H_A + H_D = 0$$

$$\frac{M_{AB} + M_{BA}}{4} + \frac{M_{CD} + M_{DC}}{6} = 0$$

$$3(M_{AB} + M_{BA}) + 2(M_{CD} + M_{DC}) = 0$$

Solve All the 3 equations and find out the values of

$$EI\Delta = \underline{23.842}$$

$$EI\theta_C = \underline{-60.127}$$

$$EI\theta_B = \underline{72.43}$$

Step - 3

- Plot B·M diagram using Moment Eqn's.

$M_{AB} = \underline{27.274}$ kN-m

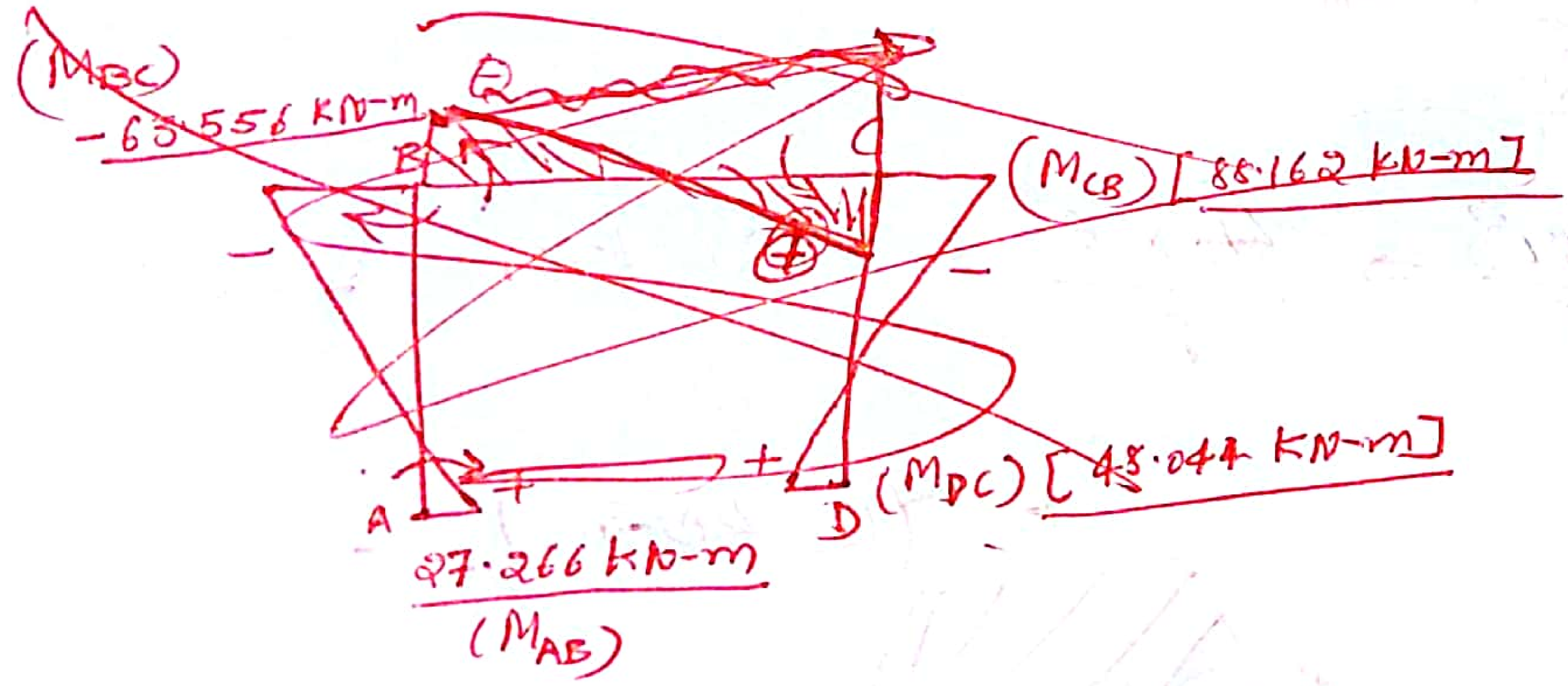
$M_{BC} = \underline{-63.556}$ kN-m

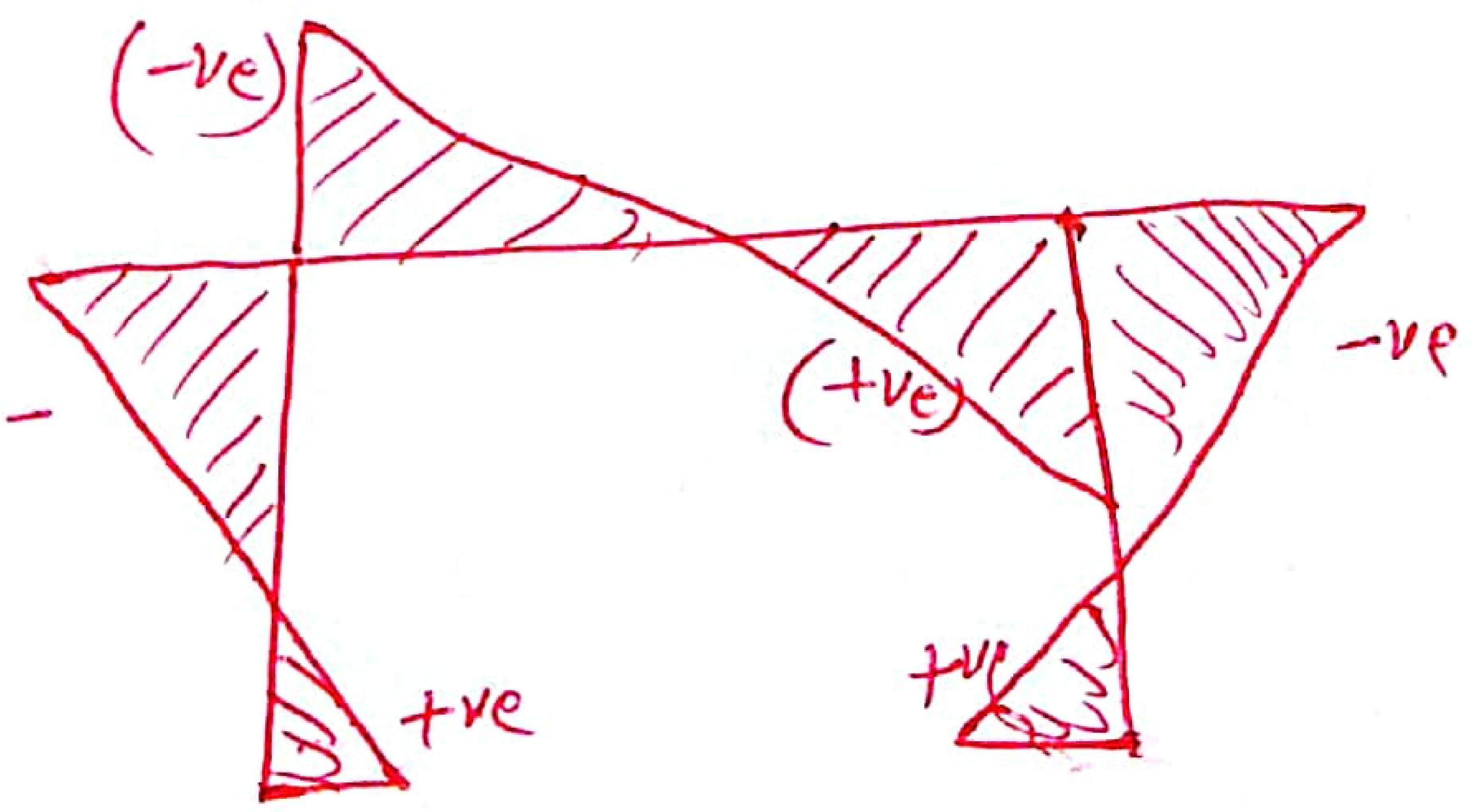
$M_{CD} = \underline{-88.162}$ kN-m

$M_{BA} = \underline{63.489}$ kN-m

$M_{CB} = \underline{88.162}$ kN-m

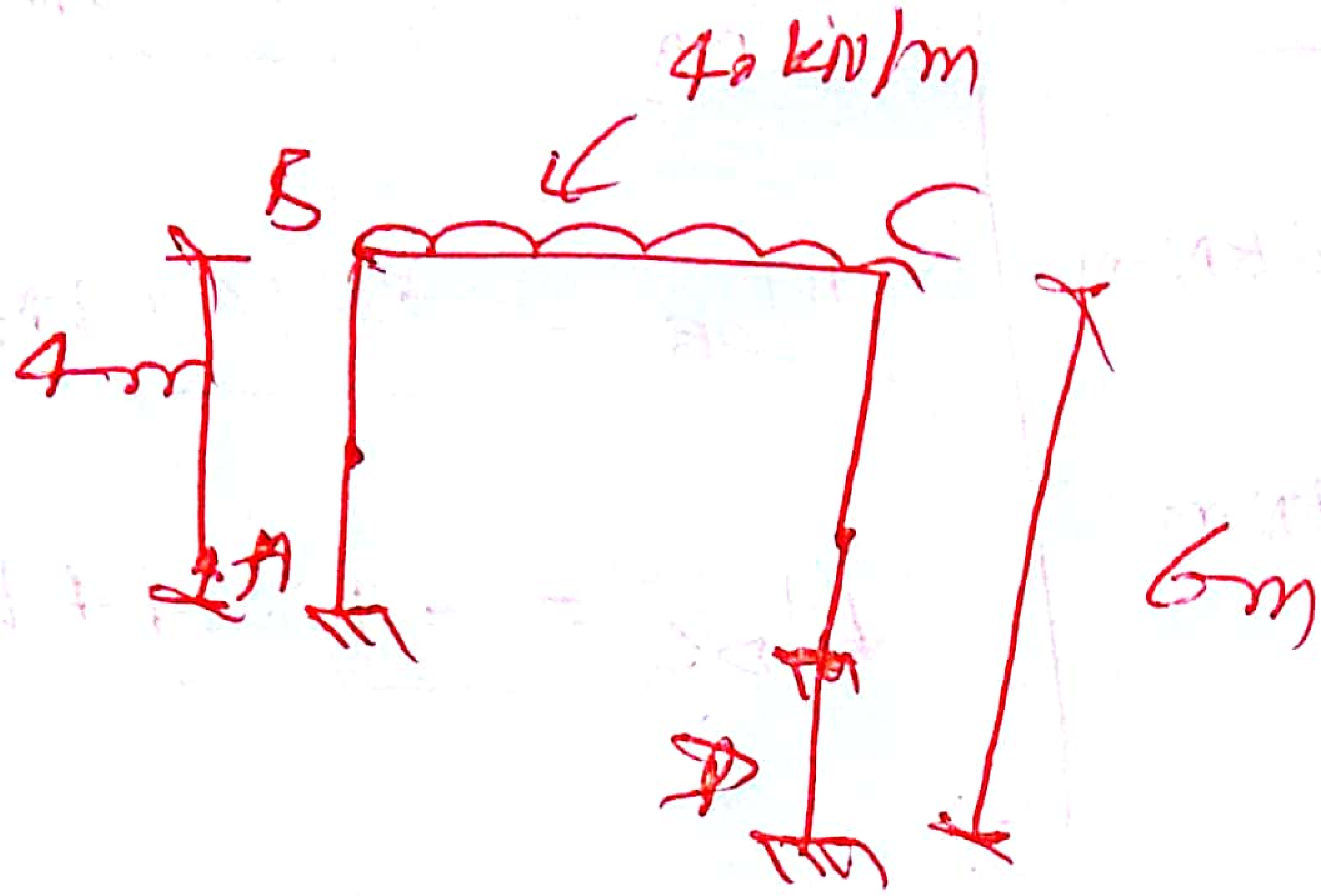
$M_{DC} = \underline{-48.044}$ kN-m



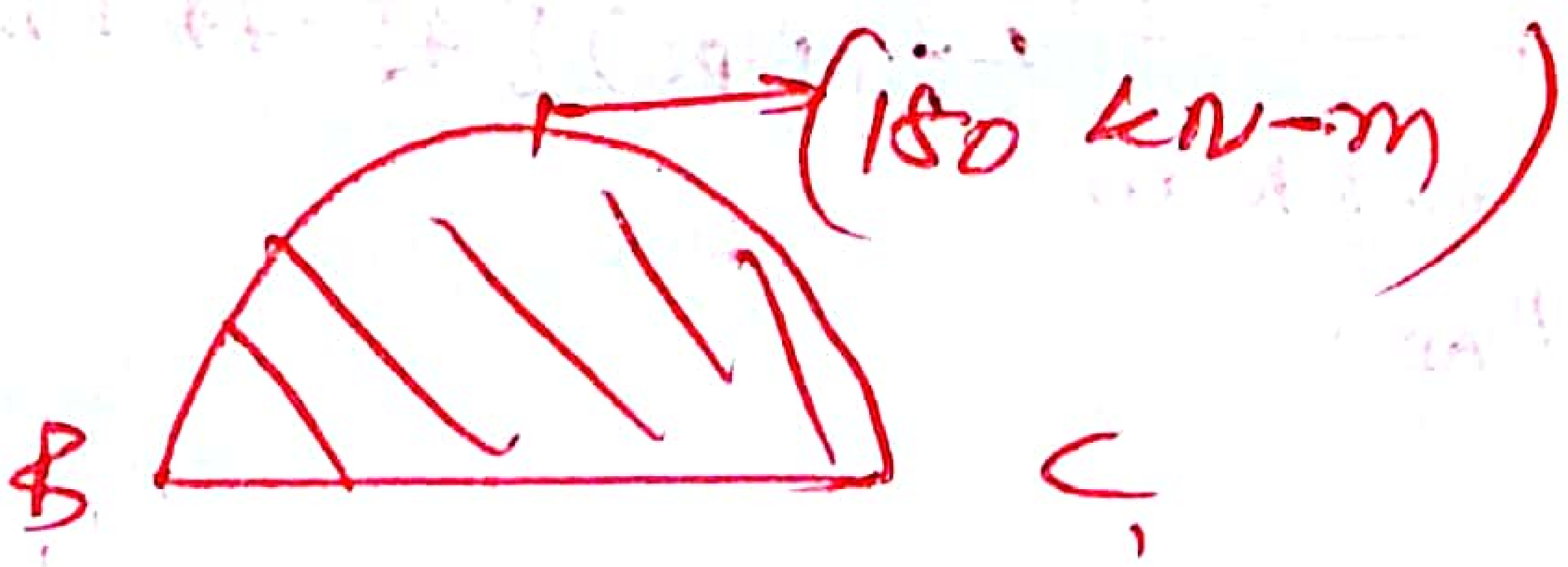


Step-4

- Draw the Bending Moment diagram As per the loading conditions



$$\frac{wL^2}{8} \Rightarrow \frac{40 \times 6^2}{8} = 180 \text{ kNm}$$



Step-5

- Overlap Both the B.M.D in Step (3) & Step (4).